Polarized parton distributions in perturbative QCD

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We review the main results of next-to-leading order QCD analyses of polarized deep-inelastic scattering data, with special attention to the assessment of theoretical uncertainties.

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1.

Experimental information on deeply inelastic scattering (DIS) of polarized leptons off polarized nucleon targets has greatly improved since the first E80/E130 experiments at SLAC (see ref. [1] for a complete bibliography.) While at an early stage the attention was mainly focused on the test of the Ellis-Jaffe [2] sum rule, more recently the interest is concentrated on the study of the general features of polarized nucleons in the deep-inelastic region in the context of the QCD-improved parton model. The analysis of polarized DIS data can at present be performed using perturbative QCD at next-to-leading order accuracy, thanks to the recent computation [3] of order α_S^2 Altarelli-Parisi splitting functions in the polarized case. This analysis has been performed by many different authors [1],[4]-[10] with consistent results. I will present here the results obtained in ref. [1]. The strategy is the same as the one adopted in the case of unpolarized DIS: the polarized parton distributions $\Delta q(x, Q^2), \Delta g(x, Q^2)$ at an initial scale Q_0 are assumed to have an arbitrarily chosen x dependence, specified by a set of unknown parameters; with the help of the QCD-improved parton model formulas and of Altarelli-Parisi evolution, one computes the structure function $g_1(x, Q^2)$ at each data point in the (x, Q^2) plane, and fits the unknown parameters, which are in turn related to interesting physical quantities.

A first important point is the test of the Bjorken sum rule [11]. The combination

$$\Gamma_{Bj} \equiv \int_0^1 dx \left[g_1^p(x, Q^2) - g_1^n(x, Q^2) \right] \tag{1}$$

can be shown to be proportional to the axial charge

$$g_A = \int_0^1 dx \left[\Delta u(x, Q^2) - \Delta d(x, Q^2) \right], \tag{2}$$

which is Q^2-independent because of current conservation, times a Wilson coefficient $C_{NS}(\alpha_S)$, which is perturbatively computable, and known to order α_S^3. This is an important and accurate theoretical prediction, since corrections to it may only come from

isospin violation ($\sim 1\%$), from terms of order α_S^4 or higher in the Wilson coefficient, or from non-perturbative contributions, suppressed by powers of Λ_{QCD}^2/Q^2. A direct test of the Bjorken sum rule has become possible, since g_1 data with deuteron and neutron targets are available. This is done by using the non-singlet axial charge g_A as one of the parameters of the fitting procedure. The result [1] is

$$g_A = 1.18 \pm 0.05(\text{exp}) \pm 0.07(\text{th}) = 1.18 \pm 0.09, \tag{3}$$

to be compared with the value $g_A = 1.257 \pm 0.003$ measured in β decay (we will discuss in the next section the unceretainties in eq. (3). It can be concluded that polarized DIS data are consistent with the Bjorken sum rule at the level of one standard deviation, with an accuracy of less than 10 %.

A second interesting question is the singlet contribution to the first moment of g_1:

$$\left[\int_0^1 dx g_1(x, Q^2)\right]_{singlet} = C_S^{(1)}(\alpha_S) a_0(Q^2), \tag{4}$$

where $C_S^{(1)}(\alpha_S)$ is the first moment of the singlet coefficient function, and $a_0(Q^2)$ the singlet axial charge; a_0 is *not* scale independent because of the axial current anomaly. In the QCD-improved parton model, one can choose the factorization scheme [10] so that the first moment of the singlet combination of polarized quark densities, $\Delta\Sigma(1)$, is scale independent, and can therefore be interpreted as the total helicity carried by quarks. In this class of schemes one has

$$a_0(Q^2) = \Delta\Sigma(1) - n_f \frac{\alpha_s(Q^2)}{2\pi} \Delta g(1, Q^2), \tag{5}$$

where $\Delta g(1, Q^2)$ is the first moment of the polarized gluon density. The values of a_0, $\Delta\Sigma(1)$ and $\Delta g(1, Q^2)$ can be extracted from the fitting procedure outlined above. We find

$$
\begin{aligned}
\Delta\Sigma(1) &= 0.46 \pm 0.04 \text{ (exp)} \pm 0.08 \text{ (th)} = 0.46 \pm 0.09, \\
\Delta g(1, 1\,\text{GeV}^2) &= 1.6 \pm 0.4 \text{ (exp)} \pm 0.8 \text{ (th)} = 1.6 \pm 0.9, \\
a_0(\infty) &= 0.10 \pm 0.05 \text{ (exp)}^{+0.17}_{-0.10} \text{ (th)} = 0.10^{+0.17}_{-0.11}.
\end{aligned}
\tag{6}
$$

The results in eqs. (6) show that large values of $\Delta\Sigma(1)$ are compatible with small values of a_0, provided $\Delta g(1, Q^2)$ is positive and large enough, as first suggested in refs. [12].

Finally, one can attempt using the value of α_S as one of the parameters in the fit [7], as customary in unpolarized data analyses. It is interesting to note that the value obtained with polarized DIS data, namely

$$\alpha_S(m_Z) = 0.120^{+0.004}_{-0.005}(\text{exp})^{+0.009}_{-0.006}(\text{th}) = 0.120^{+0.010}_{-0.008}, \tag{7}$$

is very close to other determinations, and that the uncertainty is reasonably small.

2.

We come now to a discussion of the theoretical uncertainties attached to the observables mentioned above, and summarized in table 1. This analysis has been presented in ref. [7],

	g_A	$\Delta\Sigma$	Δg	a_0	α_s
experimental	±0.05	±0.04	±0.4	±0.05	$^{+0.004}_{-0.005}$
fitting	±0.05	±0.05	±0.5	±0.07	±0.001
α_s & a_8	±0.03	±0.01	±0.2	±0.02	±0.000
thresholds	±0.02	±0.05	±0.1	±0.01	±0.003
higher orders	±0.03	±0.04	±0.6	$^{+0.15}_{-0.07}$	$^{+0.007}_{-0.004}$
higher twists	±0.03	-	-	-	±0.004
theoretical	±0.07	±0.08	±0.8	$^{+0.17}_{-0.010}$	$^{+0.009}_{-0.006}$

Table 1

Contributions to the errors in the determination of the quantities g_A, $\Delta\Sigma(1)$, $\Delta g(1,1\mathrm{GeV}^2)$, $a_0(\infty)$ and $\alpha_s(m_Z)$ from the fits described in the text.

where the interested reader can find more details. The experimental error is taken into account by the minimum-square fitting procedure. We have added in quadrature systematic and statistic errors on each data point; this procedure results probably in an overestimate of the effective uncertainty on the fit parameters, since it does not account for correlations among systematics.

A source of theoretical uncertainty which is often neglected is the arbitrariness in the choice of the functional form in x for the parton densities at the initial scale. We have considered a wide range of functional forms (see ref. [7] for details), and we have found the choice of the initial-scale parametrization affects consideralby the final results. In fact, different parametrizations lead to different estimates of contribution to the first moment of g_1 from the small-x region, where the experimental information is very poor. The corresponding spread in the determination of physical quantities must be included in the total uncertainty. This is illustrated in fig. 1, where the different curves refer to different parametrizations of the initial scale parton densities. The curves are quite close to each other in the measured region, as

Figure 1: Plot of $g_1^P(x, Q^2)$ at $Q^2 = 10$ GeV^2 for different parton density parametrizations (labelled A–D).

expected, since they all correspond to fits of comparable quality, while they differ considerably below $x \sim 0.01$. Perhaps, part of this uncertainty could be reduced using positivity constraints [13].
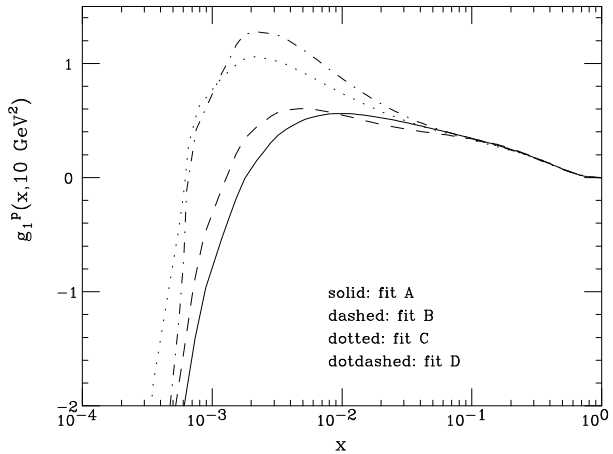
Our analysis includes data points at Q^2 down to 1 GeV^2, in order to have a reasonable information at small values of x. At such low scales, one should worry about the uncertainty originated by higher orders in the perturbative expansion. These can be estimated

by varying the values of renormalization and factorization scales μ_R, μ_F independently around Q^2. Not surprisingly, this turns out to be the most important origin of theoretical uncertainty. It should be poited out that this "theoretical" uncertainty could eventually be removed, if more data at small x and higher Q^2 were available; in fact, one could then exclude from the analysis data points below, say, $Q^2 \sim 4 - 5 \text{ GeV}^2$, as in most unpolarized DIS analyses, thus avoiding the region where the QCD perturbative expansion is less reliable.

Non-perturbative contributions are also potentially large at this low values of Q^2, since they have the form of powers of Λ^2_{QCD}/Q^2. Unfortunately, they are very difficult to estimate. One possible strategy is that of comparing results obtained fitting all data above $Q^2 = 1 \text{ GeV}^2$ with those obtained by excluding data points below 2 GeV^2. This procedure indicates that the contribution of power-suppressed terms is not very large, compared to other sources of uncertainty. A similar conclusion is obtained by fitting the Bjorken sum to its perturbative expression, supplemented with a twist-4 term a/Q^2, with the parameter a taken from renormalon and sum rule estimates.

Other minor sources uncertainties, such as violations of the $SU(3)$ flavour symmetry or the position of heavy quark thresholds in Q^2 evolution, are also listed in table 1.

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